143. Proposed by L. C. WALKER, A. M., Petaluma High School, Petaluma, Cal.

Find the area of greatest ellipse that can be inscribed in a given semicircle.

Solution by G. B. M. ZERR, A. M., Ph. D., The Temple College, Philadelphia, Pa.; J. SCHEFLER, A. M., Hagerstown, Md.; and the PROPOSER.

Let the equation of the ellipse be

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1\ldots.(1),$$

and of the circle, $x^2+(y+b)^2=r^2\ldots.(2)$.

By eliminating x^2 from (1) and (2), then writing the condition for equal roots, we have

$$b^2=a^2-\frac{a^4}{r^2}\ldots.(3).$$

Required area$=\pi ab$, or say $u=a^2b^2=a^4-(a^6/r^2)$. Hence for a maximum, we get

$$4a^3=\frac{6a^5}{r^2},\text{ or } a^2=\frac{2r^2}{3}.\quad\text{Then the area}=\frac{2\sqrt{3}}{9}\pi r^2.$$

Also solved by *JOSIAH H. DRUMMOND*.

MECHANICS.

138. Proposed by W. J. GREENSTREET, M. A., Editor of The Mathematical Gazette, Stroud, Gloucestershire, England.

A smooth elliptical tube is held in the vertical plane with its major axis inclined to the vertical. A particle is projected from the lowest point. Find the pressure on the tube at any point and the condition that the pressure may vanish at the highest point.

Solution by G. B. M. ZERR, A. M., Ph.D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa.

The reaction on the tube is given by

$$v^2/\rho=R+X\rho(d^2x/ds^2)+Y\rho(d^2y/ds^2)=R+X(dy/ds)+Y(dx/ds).$$

Let the x-axis be vertical and let β be the inclination of the major axis to the vertical; then

$$(a^2\sin^2\beta+b^2\cos^2\beta)x^2+(a^2\cos^2\beta+b^2\sin^2\beta)y^2+2(a^2-b^2)xy\sin\beta\cos\beta=a^2b^2$$

or $Ax^2+By^2+2Cxy=a^2b^2$, is the equation to the tube.

But $x=r\cos\theta$, $y=r\sin\theta$.

$$\therefore r=\frac{ab}{\sqrt{(A\cos^2\theta+B\sin^2\theta+2C\sin\theta\cos\theta)}}$$